GIAGEDU PREMIUM

SS3 PHYSICS — STRUCTURED NOTES

Quantum Theory of Radiation, the Photoelectric Effect & Wave–Particle Duality

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Structured to align with the WAEC, NECO and UTME Physics syllabus

Companion notes for the 20-slide GiagEdu Premium SS3 Physics lesson

Constants used throughout: $h = 6.63\times10^{-34}$ Js, $c = 3.0\times10^{8}$ m/s, $e = 1.6\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ kg

1. Energy Quantization and Planck's Hypothesis

Classical (wave) physics could not explain the observed spectrum of radiation emitted by a hot body (black-body radiation) — a failure historically called the “ultraviolet catastrophe.”

In 1900, Max Planck proposed that energy is not emitted or absorbed continuously, but in discrete, indivisible packets called quanta. This idea is known as Planck's Quantum Hypothesis, and the process is called energy quantization.

Key Point

- Energy exists in discrete steps (quanta), not as a continuous stream.
- This hypothesis laid the foundation for the whole of quantum theory.

2. Quantum of Energy and E = hf

Each quantum (packet) of energy associated with electromagnetic radiation of frequency f is given by Planck's equation:

$$E = hf$$

where E = energy of one quantum (J), h = Planck's constant (6.63×10^{-34} Js), and f = frequency of the radiation (Hz).

- Energy is directly proportional to frequency — higher frequency radiation carries more energy per quantum.

3. The Photon Concept

In 1905, Einstein extended Planck's idea further: he proposed that light itself travels and is exchanged in discrete packets called photons, not as a continuous wave when interacting with matter.

A photon is a massless “particle” of electromagnetic radiation that carries energy E = hf and travels at the speed of light, c.

- Photons have energy and momentum but no rest mass.
- The photon concept explains why light can knock electrons out of a metal (see Section 5).

4. Photon Energy and Wavelength Calculations

Since the wave equation gives $c = f\lambda$, frequency can be written as $f = c/\lambda$. Substituting into $E = hf$ gives photon energy in terms of wavelength:

$$E = hf = hc / \lambda$$

This shows that photon energy is inversely proportional to wavelength: shorter-wavelength radiation (e.g. violet light, UV, X-rays) carries more energetic photons than longer-wavelength radiation (e.g. red light, infrared).

Worked Example A

Calculate the energy of a photon of wavelength 500 nm.

$E = hc/\lambda = (6.63\times10^{-34} \times 3.0\times10^{8}) / (500\times10^{-9}) = 3.98\times10^{-19}$ J (≈ 2.49 eV)

5. The Photoelectric Effect

The photoelectric effect is the emission of electrons (photoelectrons) from the surface of a metal when electromagnetic radiation of sufficiently high frequency falls on it. It was first observed by Hertz (1887) and explained theoretically by Einstein (1905).

Key Experimental Observations

- No photoelectrons are emitted if the frequency of the incident light is below a certain minimum value (the threshold frequency), no matter how intense the light is.
- Above the threshold frequency, emission of photoelectrons is instantaneous — there is no time delay.
- The maximum kinetic energy of the emitted photoelectrons depends on the frequency of the incident light, not on its intensity.
- The number of photoelectrons emitted per second (photocurrent) depends on the intensity of the incident light.

These observations could not be explained by the classical wave theory of light, which predicted that emission should depend on intensity, not frequency — this failure led directly to Einstein's photon-based explanation.

6. Work Function

The work function (symbol W_o or Φ) of a metal is the minimum amount of energy required to release (eject) an electron from the surface of that metal.

$$W_o = hf_o$$

The work function is measured in joules (J) or electron-volts (eV), where 1 eV = 1.6×10^{-19} J. Work function varies from metal to metal — for example, sodium (≈ 2.3 eV) has a much lower work function than platinum (≈ 6.35 eV).

7. Threshold Frequency and Threshold Wavelength

The threshold frequency (f_o) is the minimum frequency of incident light that will just eject electrons from a given metal surface (with zero kinetic energy). Below this frequency, no photoelectrons are emitted, regardless of intensity.

$$f_o = W_o / h$$
$$\lambda_o = c / f_o = hc / W_o$$

λ_o (the threshold wavelength) is the longest wavelength of light that can still cause photoemission from the metal. Any light with $\lambda > \lambda_o$ (i.e. $f < f_o$) cannot eject electrons no matter how bright it is.

Worked Example B

The work function of sodium is 2.3 eV. Find its threshold frequency and threshold wavelength.

$W_o = 2.3 \times 1.6\times10^{-19} = 3.68\times10^{-19}$ J

$f_o = W_o/h = 3.68\times10^{-19} / 6.63\times10^{-34} = 5.55\times10^{14}$ Hz

$\lambda_o = c/f_o = 3.0\times10^{8} / 5.55\times10^{14} = 5.41\times10^{-7}$ m (≈ 541 nm)

8. Stopping Potential

The stopping potential (V_o or Vs) is the minimum reverse (retarding) potential difference that must be applied between the metal surface and the collector to just stop even the most energetic photoelectrons from reaching the collector — reducing the photocurrent to zero.

$$eV_o = KE_{max}$$

where e is the charge on an electron (1.6×10^{-19} C). The stopping potential gives a direct, measurable way of finding the maximum kinetic energy of photoelectrons in the laboratory.

9. Einstein's Photoelectric Equation

Einstein combined the photon concept with the idea of the work function to give a single equation that fully explains the photoelectric effect: the energy of an incoming photon is used partly to free the electron (the work function) and partly becomes the electron's kinetic energy.

$$hf = W_o + KE_{max}$$
$$KE_{max} = hf - W_o = h(f - f_o) = eV_o$$

This single equation correctly accounts for every observation listed in Section 5: it shows why emission depends on frequency (not intensity), why there is a threshold frequency, and why emission is instantaneous.

10. Wave Nature of Light

Certain phenomena can only be explained if light behaves as a wave:

- Interference — e.g. Young's double-slit experiment, producing bright and dark fringes.
- Diffraction — the bending of light around obstacles and through narrow slits.
- Polarization — light can be restricted to vibrate in a single plane, a property only waves possess.

As a wave, light has a wavelength (λ) and frequency (f), related by $c = f\lambda$, where c is the speed of light.

11. Particle Nature of Light

Other phenomena can only be explained if light behaves as a stream of particles (photons):

- The photoelectric effect — explained in Sections 5–9.
- The Compton effect — the scattering of X-ray photons off electrons, with a measurable change in wavelength consistent with particle-like collisions.

As particles, photons carry quantized energy $E = hf$ and momentum $p = h/\lambda$, and interact with matter as discrete “bundles” rather than a spread-out wave.

12. Wave–Particle Duality (The Paradox)

Light exhibits wave properties (interference, diffraction, polarization) in some experiments, and particle properties (photoelectric effect, Compton scattering) in others. Neither the pure wave model nor the pure particle model, on its own, can account for all observed behaviour of light — this apparent contradiction is called the wave–particle paradox, or wave–particle duality.

Modern quantum mechanics resolves the paradox by treating wave and particle behaviour as complementary descriptions of the same physical reality: which behaviour is observed depends on the type of experiment being performed.

13. De Broglie Hypothesis

In 1924, Louis de Broglie proposed a bold extension of wave–particle duality: if light (a wave) can behave like particles (photons), then particles of matter (such as electrons) should also exhibit wave-like behaviour.

Every moving particle therefore has an associated wavelength, called its de Broglie wavelength. This hypothesis was later confirmed experimentally by the Davisson–Germer electron diffraction experiment, which showed electrons producing diffraction patterns just like waves.

14. The de Broglie Wavelength: λ = h/p = h/mv

The de Broglie wavelength of a moving particle is given by:

$$\lambda = h / p = h / mv$$

where p is the momentum of the particle, m is its mass, and v is its velocity. Note that for everyday (macroscopic) objects, m is so large that λ is immeasurably small; the wave nature of matter only becomes noticeable for very small particles such as electrons.

Worked Example C

Calculate the de Broglie wavelength of an electron (mass 9.11×10^{-31} kg) moving at 2.0×10^{6} **m/s.**

$p = mv = 9.11\times10^{-31} \times 2.0\times10^{6} = 1.82\times10^{-24}$ kg·m/s

$\lambda = h/p = 6.63\times10^{-34} / 1.82\times10^{-24} = 3.64\times10^{-10}$ m (≈ 0.364 nm — comparable to atomic spacing, which is why electrons diffract through crystals)

15. Worked Numerical Examples

The following examples bring several formulas together, in the style commonly tested by WAEC, NECO and UTME.

Worked Example 1

Light of wavelength 400 nm is incident on a metal of work function 2.0 eV. Calculate the maximum kinetic energy of the emitted photoelectrons.

Photon energy: $E = hc/\lambda = (6.63\times10^{-34} \times 3.0\times10^{8}) / (400\times10^{-9}) = 4.97\times10^{-19}$ J = 3.11 eV

$KE_{max} = E - W_o = 3.11 - 2.0 = 1.11$ eV (= 1.78×10^{-19} J)

Worked Example 2

The threshold wavelength of a metal is 620 nm. Calculate (i) the work function in joules and eV, and (ii) the threshold frequency.

$f_o = c/\lambda_o = 3.0\times10^{8} / 620\times10^{-9} = 4.84\times10^{14}$ Hz

$W_o = hf_o = 6.63\times10^{-34} \times 4.84\times10^{14} = 3.21\times10^{-19}$ J (= 2.01 eV)

Worked Example 3

Calculate the momentum and stopping potential associated with a photoelectron of maximum kinetic energy 1.6×10^{-19} **J.**

Momentum: $p = \sqrt{(2m\cdot KEmax)} = \sqrt{(2 \times 9.11\times10^{-31} \times 1.6\times10^{-19})} = \sqrt{(2.92\times10^{-49})} = 5.40\times10^{-25}$ kg·m/s

$V_o = KE_{max}/e = 1.6\times10^{-19} / 1.6\times10^{-19} = 1.0$ V

16. WAEC / NECO / UTME Examination-Style Questions

Section A: Objective Questions

1. The phenomenon in which electrons are emitted from a metal surface when light of suitable frequency falls on it is called:

A. Thermionic emission B. Photoelectric effect C. Compton effect D. Field emission

2. Planck's constant has the approximate value:

A. 6.63×10^{-23} Js B. 6.63×10^{-34} Js C. 3.0×10^{8} Js D. 1.6×10^{-19} Js

3. The minimum energy required to eject an electron from a metal surface is called the:

A. Ionization energy B. Binding energy C. Work function D. Excitation energy

4. Einstein's photoelectric equation is:

A. $E = mc^2$ B. $hf = W_o + KE_max$ C. $\lambda = h/mv$ D. $E = hf/2$

5. If the frequency of incident light is less than the threshold frequency, then:

A. Electrons are emitted slowly B. No electrons are emitted C. Electrons are emitted with high KE D. Electrons are emitted after a delay

6. The wave nature of light is best demonstrated by:

A. The photoelectric effect B. The Compton effect C. Interference and diffraction D. Emission of photons

7. According to de Broglie, a particle of momentum p has an associated wavelength given by:

A. $\lambda = hp$ B. $\lambda = h/p$ C. $\lambda = p/h$ D. $\lambda = hp^2$

8. The stopping potential in a photoelectric experiment is a direct measure of the:

A. Work function B. Threshold frequency C. Maximum kinetic energy of photoelectrons D. Intensity of the incident light

9. Which of the following correctly relates the energy and wavelength of a photon?

A. $E = h\lambda$ B. $E = hc/\lambda$ C. $E = h/\lambda$ D. $E = hc\lambda$

10. An electron accelerated through a potential difference V gains kinetic energy equal to:

A. eV B. e/V C. V/e D. e^2V

Answers to Section A

1.B 2.B 3.C 4.B 5.B 6.C 7.B 8.C 9.B 10.A

Section B: Theory Questions

1. (a) State Einstein's photoelectric equation, explaining each term.

(b) Light of wavelength 400 nm falls on a metal surface of work function 2.0 eV. Calculate the maximum kinetic energy of the emitted photoelectrons. (See Worked Example 1 for the full solution method.)

2. (a) Distinguish between the wave nature and particle nature of light, citing one experimental piece of evidence for each.

(b) State de Broglie's hypothesis and write the formula for the de Broglie wavelength of a particle of mass m moving with velocity v.

3. (a) Define (i) threshold frequency and (ii) stopping potential.

(b) The threshold wavelength of a metal is 620 nm. Calculate its work function in joules and in eV, and its threshold frequency. (See Worked Example 2.)

17. Summary and Homework

Key Formulas at a Glance

- Photon energy: $E = hf = hc/\lambda$
- Work function: $W_o = hf_o$
- Threshold wavelength: $\lambda_o = hc/W_o$
- Einstein's photoelectric equation: $hf = W_0 + KE_{max}$
- Stopping potential: $eV_0 = KE_{max}$
- De Broglie wavelength: $\lambda = h/p = h/mv$

Summary of Key Ideas

- Energy is quantized (Planck) and light is exchanged in discrete photons (Einstein).
- The photoelectric effect shows that light behaves as particles; interference and diffraction show that light behaves as a wave — this is wave–particle duality.
- De Broglie extended this duality to matter: moving particles also have an associated wavelength.
- Einstein's photoelectric equation, $hf = W_0 + KE_{max}$, ties together frequency, work function and photoelectron kinetic energy, and is central to WAEC/NECO/UTME numerical questions.

Homework

- 1. State Planck's quantum hypothesis in your own words.
- 2. Calculate the energy of a photon of frequency 6.0×10^{14} Hz.
- 3. The work function of a metal is 4.5×10^{-19} J. Find its threshold frequency and threshold wavelength.
- 4. Light of wavelength 300 nm falls on a metal of work function 2.5 eV. Calculate the maximum kinetic energy of the photoelectrons and the stopping potential.
- 5. Explain why the classical wave theory of light could not account for the instantaneous emission of photoelectrons.
- 6. Calculate the de Broglie wavelength of a proton (mass 1.67×10^{-27} kg) moving at 1.0×10^{5} m/s.
- 7. State two similarities and two differences between a photon and an electron.
- 8. With the aid of a labelled diagram, describe an experiment to demonstrate the photoelectric effect and explain how the stopping potential is measured.